| SUBJECTS: |
| :--- |
| Math (Advanced), Science (Physics) |
| TIME: |
| 1 class period |
| MATERIALS: |
| calculator |
| paper |
| pencil |
| student sheet and figures |

## OBJECTIVES

The student will do the following:

1. Compute math problems dealing with volume of groundwater.
2. Calculate average porosity in one type of aquifer.
3. Calculate the volume of water that will supply a particular well and its recharge rate.

## BACKGROUND INFORMATION

Groundwater accounts for a major portion of the world's freshwater resources. Estimates of the global water supply show groundwater as 0.6 percent of the world's total water and 60 percent of the available fresh water resources. The total volume of readily available global groundwater is about $4.2 \times 106 \mathrm{~km} 3$ as compared to $0.126 \times 106 \mathrm{~km} 3$ (kilometers cubed) stored in lakes and streams. Next to glaciers and icecaps, which do not have readily available water, groundwater reservoirs are the largest holding basins for fresh water in the world hydrologic cycle (Figure 1).

The age of groundwater may range from a few years or less to tens of thousands of years or more. For the United States, it is estimated that about 25 percent of precipitation becomes groundwater.

It is estimated that the total usable groundwater in storage is about equivalent to the total precipitation for ten years, or the total surface runoff to streams and lakes for 35 years, although all of this groundwater is not available for practical use. In the United States, groundwater storage exceeds by many times the capacity of all surface reservoirs and lakes, including the Great Lakes.

Recoverable groundwater is that water released from storage in the subsurface zone of saturation whose capacity is the total volume of the pores or openings in soil or rocks that are filled with water.
The porosity values of specific materials are shown in Figure 2.
Groundwater movement is dependent on the degree of interconnection of the porous space (permeability) and the gradient or slope of the water table. These factors vary greatly depending on the aquifer type. Groundwater in a carbonate aquifer can occasionally move through limestone caverns as rapidly as surface water ( $1-3 \mathrm{ft} / \mathrm{sec}$ ). In sandy aquifers, groundwater can move as slowly as $3 \mathrm{ft} / \mathrm{day}$ or even as slowly as $1 \mathrm{in} /$ day. For those reasons, groundwater cleanup, or remediation, can only be done by enhanced methods.* These methods are complex and expensive, making groundwater pollution prevention quite economical. There are obvious health protection reasons for groundwater pollution prevention, as well.

## Terms:

## carbonate aquifer:

underground layer of limestone that is saturated with usable amounts of water

## gradient:

change of elevation, velocity, pressure, or other characteristics per unit length; slope

## hydrologic cycle:

the cyclical process of water's movement from the atmosphere, its inflow and temporary storage on and in land, and its outflow to the oceans; cycle of water from the atmosphere by condensation, and precipitation, then its return to the atmosphere by evaporation and transpiration.

## permeability:

the capacity of a porous material to transmit fluids. Permeability is a function of the sizes, shapes, and degree of connection among pore spaces, the viscosity of the fluid, and the pressure driving the fluid.
porosity:
the spaces in rock or soil not occupied by solid matter.

## water table:

upper surface of the zone of saturation of groundwater

## ADVANCE PREPARATION

A. Copy Figures 1 and 2, Student Sheets, and Activity Section for students.
B. Complete the math problems before students are given them. (See the teacher sheet.)

* For information on enhanced methods, see activity "Groundwater: Cleaning Up" in this chapter.


## PROCEDURE

## I. Setting the stage

A. Give students Conversion Student Sheet, Figures 1 and 2, and Activity Section.
B. If students need help with the math, give them the Hint Page.

## II. Activity

Problems:
A. How many gallons are represented by $4.2 \times 106 \mathrm{~km} 3$ of global groundwater?
B. If $1 \times 106 \mathrm{mi} 3$ of subsurface water exists in a volume of the earth's crust that covers
$5 \times 107 \mathrm{mi} 2$ of the land surface land is $1 / 2$ mile deep, what is the average porosity (in \%) of the
upper layer of the Earth's crust?* (assuming that the entire depth is saturated)
C. If a recharge area of a water well measures 1000 acres and the well is 800 meters deep, and soil porosity is that of a sand and gravel mix, what volume of water might theoretically supply the well?*
D. In question C above, what is the annual recharge rate (gallons/year) if precipitation is 55
inches per year, 7 inches of which become groundwater?
*Note: Solutions to $B, C$, and $D$ are oversimplifications of these types of determinations but generally demonstrate the principles of recharge, yield, and infiltration calculations.

## RESOURCES

Bouwer, Herman, Groundwater Hydrology, McGraw Hill Book Company, New York, NY, 1978,
pp. 2-3, 6-8.
Groundwater and Wells, Johnson Division, UOP, Inc. St. Paul, MN, 1975, pp. 17-18.
Groundwater Pollution and Hydrology, Princeton University Short Course, 1983, pp. 1-2.

## The Earth's Water Resources

| SOURCE | SURFACE <br> AREA <br> (mi2) | WATER <br> VOLUME <br> $(\mathrm{mi} 3)$ | \% OF TOTAL <br> WATER |
| :--- | :--- | :--- | :--- |
| Surface water <br> freshwater lakes <br> $=1 / 2$ mi. deep | 330,000 | 30,000 | .009 |
| Saline lakes | 270,000 | 25,000 | .008 |
| Stream channels | --- | 500 | .0001 |
| Subsurface water <br> $<1 / 2$ <br> mi. deep | $50,000,000$ | $1,000,000$ | .31 |
| Subsurface water <br> $>1 / 2 ~ m i . ~ d e e p ~$ | $50,000,000$ | $1,000,000$ | .31 |
| Soil moisture + <br> water <br> in vadose zone | $50,000,000$ | 16,000 | .005 |
| Glaciers/ice caps | $6,900,000$ | $7,000,000$ | 2.15 |
| Atmosphere | $197,000,000$ | 3,100 | .001 |
| Oceans | $139,500,000$ | $317,000,000$ | 97.2 |
| TOTAL | $326,000,000$ |  |  |

Figure 1

## Porosities

## Porosities of specific materials. Approximate ranges are:

| Materials | Porosity, percentage |
| :---: | :---: |
| Silts and clays (that have not been significantly compacted) | 50-60 |
| Fine sand | 40-50 |
| Medium sand | 35-40 |
| Coarse sand | 25-35 |
| Gravel | 20-30 |
| Sand and gravel mixes | 10-30 |
| Glacial till | 25-45 |
| Dense, solid rock | <1 |
| Fractured and weathered igneous rock | 2-10 |
| Permeable, recent basalt | 2-5 |
| Vesicular lava | 10-50 |
| Tuff | 30 |
| Sandstone | 5-30 |
| Carbonate rock with original and secondary porosity | 10-20 |

Figure 2

## CONVERSIONS

$$
\begin{aligned}
& 1 \mathrm{yd} 3=27 \mathrm{ft} 3 \\
& 1 \mathrm{acre}=4047 \mathrm{~m} 2 \\
& 1 \text { hectare }=10,000 \mathrm{~m} 2 \\
& 1 \mathrm{~m}=3.28 \mathrm{ft} \\
& 1 \mathrm{acre}=43,560 \mathrm{ft} 2 \\
& 1 \mathrm{ft} 3=7.48 \mathrm{gal} \\
& 1 \mathrm{~m} 3=35.31 \mathrm{ft} 3
\end{aligned}
$$

## HINT PAGE - Use Student Sheet on Conversions.

A To start this problem, look at the conversion sheet.

1. Convert to m3.
2. Convert m 3 to ft 3 .
3. Convert ft 3 to gallons.
B.
4. Porosity $=\frac{\mathrm{mi3} \text { of } \mathrm{H} 2 \mathrm{O}}{\mathrm{mi} 3 \text { of soil }} \quad \frac{\text { (volume of } \mathrm{H} 2 \mathrm{O})}{\text { (volume of soil) }}$
5. Volume $=$ area $x$ depth.
( mi3 of soil $=m i 2$ of land $x$ depth in miles)
C.
6. Convert acres to meters.
7. Volume $=$ area $x$ depth.
8. Water yield = volume $\times$ porosity (convert to decimal).
D.
9. Convert inches to ft.
10. Convert ft to meters.
11. Convert acres to m2.
12. Multiply area $(\mathrm{m} 2) \mathrm{x}$ rainfall that becomes groundwater $(\mathrm{m})=\mathrm{m} 3$.
13. Must convert m 3 to ft 3 first.
14. Convert ft3 to gallons.

## A. To start this problem, you must look at the conversion sheet.

1. Convert to m3.
$4.2 \times 106 \mathrm{~km} 3 \times \frac{1 \times 109 \mathrm{~m} 3}{1 \mathrm{~km} 3}=4.2 \times 1015 \mathrm{~m} 3$
2. Convert m3 to ft3.

$$
\frac{4.2 \times 1015 \mathrm{m3}}{1 \mathrm{~m} 3} \times 35.31 \mathrm{ft} 3=1.483 \times 1017 \mathrm{ft} 3
$$

3. Convert ft3 to gallons.
$\frac{1.483 \times 1017 \mathrm{ft} 3}{1 \mathrm{ft} 3} \times 7.48 \mathrm{gal}=1.11 \times 1018 \mathrm{gal}$
B.
4. Porosity $=\quad \frac{\mathrm{mi3} \text { of } \mathrm{H} 2 \mathrm{O}}{\mathrm{mi} \text { of soil }} \quad \frac{(\text { volume of H2O) }}{(\text { volume of soil })}$
5. Volume $=$ area $\times$ depth.

$$
\begin{aligned}
& \mathrm{mi} 3 \text { of soil }=\mathrm{mi2} \text { of land } \times \text { depth in miles } \\
& \mathrm{mi} 3 \text { of soil }=(5 \times 107) \times 0.5=2.5 \times 107 \mathrm{mi} 3 \\
& \text { Porosity }=\frac{1 \times 106}{2.5 \times 107}=0.04 \text { or } 4 \%
\end{aligned}
$$

C.

1. Convert acres to meters.

$$
1000 \text { acres } \times \frac{4074 \mathrm{~m} 2}{1 \text { acre }}=4.074 \times 106 \mathrm{~m} 2
$$

2. Volume $=$ area $x$ depth.
$4.074 \times 106 \mathrm{~m} 2 \times 800 \mathrm{~m}=3.26 \times 109 \mathrm{~m} 3$
3. Water yield = volume $\times$ porosity (convert to decimal).
$3.26 \times 109 \mathrm{~m} 3 \times 0.20=6.52 \times 108 \mathrm{~m} 3$
D.
4. Convert inches to ft.

7 in $x \frac{1 \mathrm{ft}}{12 \mathrm{in}}=0.58 \mathrm{ft}$
2. Convert ft to meters.
$0.58 \mathrm{ft} \times \frac{1 \mathrm{~m}}{3.28 \mathrm{ft}}=0.177 \mathrm{~m}$ of rain into groundwater
3. Convert acres to m2.

1000 acres $\times \frac{4047 \mathrm{~m} 2}{1 \text { acre }}=4.047 \times 106 \mathrm{~m} 2$
4. Multiply area (m2) $x$ rainfall that becomes groundwater $(m)=m 3$.

$$
4.047 \times 106 \mathrm{~m} 2 \times 0.177 \mathrm{~m}=7.16 \times 105 \mathrm{~m} 3
$$

5. Must convert m3 to ft3 first.

$$
7.16 \times 105 \mathrm{~m} 3 \times \frac{35.31 \mathrm{ft} 3}{1 \mathrm{~m} 3}=2.53 \times 107 \mathrm{ft} 3
$$

6. Convert ft3 to gallons.
$2.53 \times 107 \mathrm{ft} 3 \times \frac{7.48 \mathrm{gal}}{1 \mathrm{ft} 3}=1.89 \times 108 \mathrm{gal}$
